A Generalized Horn Design to Optimize Directivity Control & Wavefront Curvature

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Charles E. Hughes
Peavey Electronics
Meridian, MS, USA

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Meridian, MS

A new horn design is presented. This approach yields good loading characteristics and reduced harmonic distortion. The new horns polar patterns are that of a constant directivity type horn. The novel feature of this new horn is that its apparent apices for the horizontal and vertical planes are in the same physical location regardless of coverage angle for the horizontal or vertical plane.

0. Introduction

Horns have been used in loudspeaker systems for decades. Their use can be attributed to two main factors: 1) increased output for a given input and 2) directivity control. The latter has been seen to be the more dominant reason in recent years. A good degree of loading of the loudspeaker driver must still be accomplished for the horn to be useful. However, with higher power handling and increased efficiency available from typical compression drivers today, horn designers can concentrate more on the directivity response of a horn.

In recent years Geddes[1] and Putland[2] have put forth works on horns that have the properties of propagating a one-parameter acoustic wave. The motion of such a wave can be described by a single spatial coordinate. These types of horns have been referred to as waveguides. A good differentiation between a horn and a waveguide can be thought of as a horn being primarily concerned with the optimal loading of its driver, while a waveguide is primarily concerned with its directional characteristics[3].

There are a several terms that will be used later. Their definition is as follows.

$r \equiv$ radius of connecting arc

$y_t \equiv$ diameter of throat entrance

$r_H \equiv$ radius of horizontal plane connecting arc

$r_V \equiv$ radius of vertical plane connecting arc

$\theta_H \equiv$ horizontal coverage half - angle

$\theta_V \equiv$ vertical coverage half - angle

1. Constant Directivity

In 1975 Keele[4] outlined a concept whereby a horn could be constructed of essentially separate, but joined, sections. The first section of this type of horn has a cross sectional area that expands exponentially. The second section has a cross sectional area that expands in a manner similar to that of a simple cone. This combined horn had two very sought after traits. It presented good loading to the driver to which it was attached allowing for an increase in the efficiency of the horn. It also maintained good directivity control over a wide range of frequencies.

Up until this time the exponential horn was a mainstay of the industry. It loaded the driver to which it was attached very well. This gave it good efficiency to reasonably low frequencies. Its primary drawback was that as frequency increased, the directivity, or coverage pattern, of the horn would start to narrow. This was very undesirable because as listeners became more off-axis to the horn they would not receive as much high frequency information.

Keele's horn gave good loading and a more consistent beamwidth or directivity, hence the term Constant Directivity.

In 1977 Henricksen & Ureda[5] introduced what they called the Manta-Ray horn. It was named for its shape. This horn had good loading as well as good directivity control. However, it did suffer in some areas. One in particular was that it had rather severe astigmatism in the curvature of its wave front. Put simply, the radius of curvature for the wavefront is different for the horizontal and vertical planes. Accordingly, the shape of the wavefront from such a device can, at best, be ellipsoidal. While this doesn't seem to be cause for concern when the horn is used by itself, when more than one horn is employed in an array it can be problematic. This astigmatism is common to a number of horn designs within the industry today.

In addition to the astigmatism, these types of horns suffered in other areas as well. The slowly expanding exponential section can tend to cause distortion as high sound pressure levels are reached. The discontinuities in some of the horns, where the sections are connect, can cause reflections back down the horn that can be problematic. These discontinuities also cause diffraction, which introduce another set of distortion products. Walls that are parallel, or nearly parallel, in the throat section of some of these horns can lead to unwanted resonance conditions.

2. Solving the Problem of Astigmatism

Most conventional constant directivity type horns have at least one item in common. That is the main waveguide section of the horn is conical in nature. That is to say that it has predominately straight walls. Such a horn falls into the category of admitting and propagating a one-parameter wave[2]. This is probably the reason for the constant coverage of such a device. As such, the main body of this new waveguide will also be comprised of straight walls.

A solution was sought for the astigmatic wavefront that is typical for conventional constant directivity horns. Since this problem is caused by a horn having different points from which the wavefront appears to originate in the horizontal and vertical planes, one must simply make the point the same for both coverage planes. This point was located in the center of the throat entrance to the waveguide as shown in figure 1. The challenge now was how to join the required throat diameter to the straight walls of the required design coverage angles. A circular arc turns out to perform this function quite nicely. This arc is from a point on the perimeter of the throat to the straight wall of the waveguide. The radius of the arc is such that one end of the arc is tangent to the straight wall at one point while the other end of the arc is perpendicular to the plane of the throat at the perimeter of the throat entrance. This can be seen in figure 2.

The new waveguide can be thought of as being constructed in distinctly separate, but joined sections. The first section is the throat section that is comprised of the connecting arc described above. The last section is comprised of straight walls in order to obtain the desired directional properties. Since different coverage angles are typically required for the horizontal and vertical planes, the throat section becomes subdivided into two sections. The first of these sections will be where, in both the horizontal and vertical coverage planes; the connecting arcs define the wall shapes. The second of these sections will be where only one of the coverage planes, either horizontal or vertical has a connecting arc to define its wall shape. The other coverage plane in this section has its wall shape defined by a straight wall at the design coverage angle.

A method had to be implemented to go from the circular throat entry to the rectangular cross sectional shape of the main body of the waveguide. The throat section can be made to change from circular to rectangular by allowing elliptically shaped fillets to develop in the corners. These elliptic corner fillets begin as a circle at the throat; the horizontal radius equals the vertical radius. As the length down the horn increases the corners radii become smaller so that they attain their final desired value at the point where that section terminates into the straight wall section. The final value of the corner fillet is arbitrary and can be chosen based on aesthetics or manufacturing constraints. Since the horizontal coverage angle is different from the vertical coverage angle, the horizontal radius of the corner fillet will change at a different rate than the vertical radius of the corner fillet. This is what gives rise to the elliptical nature of the fillets.

An initial prototype was constructed at this point to test the performance. The results were extremely promising. This prototype exhibited good amplitude response, good loading characteristics and very good directional characteristics.

3. Development of the Design Equations

To analyze the new design, equations describing the cross sectional area expansion within the throat section had to be developed. A relationship between known design parameters and the connecting arc, r, were also needed. To simplify matters we will only concern ourselves with the axisymmetric, or circular, case for the time being. Referring to the figure 3.

$$r^2 + a^2 = (r + \frac{y_t}{2})^2 \qquad 1 + \frac{a^2}{r^2} = \left(\frac{r + \frac{y_t}{2}}{r^2} \right)^2 \qquad 1 + \tan^2\theta = \left(\frac{r + \frac{y_t}{2}}{r^2} \right)^2$$

$$r = \frac{y_t}{2}\left(\sqrt{1 + \tan^2\theta} - 1\right) \qquad (1)$$

Equation (1) gives us the relationship between the radius of the connecting arc, r, the throat diameter, y_t, and the design coverage angle, θ.

To derive an expression for the height, or diameter, within the throat section, let h be the incremental height difference from the throat entry diameter, y_t, and the waveguide boundary.

$$y = y_t + 2h \qquad (r - h)^2 = r^2 - x^2 \qquad h = r - \sqrt{r^2 - x^2}$$

$$y = y_t + 2\left(r - \sqrt{r^2 - x^2}\right) \qquad (2)$$

Equation (2) gives us an expression for the height within the throat section of the waveguide. We may now proceed to the derivation for the cross sectional area expansion.

$$S = \pi\left(\frac{y}{2}\right)^2 = \pi\left(\frac{y_t}{2} + r - \sqrt{r^2 - x^2}\right)^2$$

This reduces to

$$S = \pi\left(x^2 - (y_t + 2r)\sqrt{r^2 - x^2} + y_t\left(\frac{y_t}{4} + r\right)\right) \qquad (3)$$

$$S = Ax^2 + B\sqrt{r^2 - x^2} + C$$

$$A = \pi \qquad B = -\pi(y_t + 2r) \qquad C = \pi y_t\left(\frac{y_t}{4} + r\right)$$

Equation (3), while not in the classical form of a quadratic equation, does posses the traits of having an x^2, an x and a constant term. As such, this new waveguide design has been dubbed a Quadratic Throat waveguide.

For the non-axisymmetric cases the expansions do not lend themselves to this level of algebraic reduction. Hence their expansion is given by

$$S = \left(y_t + 2\left(r_H - \sqrt{r_H^{\,2} - x^2}\right)\right) * \left(y_t + 2\left(r_V - \sqrt{r_V^{\,2} - x^2}\right)\right) \qquad (4)$$

From equation (1)

$$r_H = \frac{y_t}{2}\left(\sqrt{1+\tan^2\theta_H} - 1\right) \qquad\qquad r_V = \frac{y_t}{2}\left(\sqrt{1+\tan^2\theta_V} - 1\right)$$

Equation (4) is the expansion for the rectangular case that was used for the development of the waveguides presented in the paper.

4. Analysis of the New Waveguide Design

As was stated in Section 2 the loading that this new waveguide presented to its drivers seemed to be good. However, a more quantitative measure was desired. For some time now this author has been using a method of analyzing the loading of a horn that was brought to his attention by Gunness[6]. This method compares the instantaneous flare rate of an arbitrary shape horn to that of a classical exponential horns flare rate. The instantaneous flare rate is determined for any point along the length of the horn. This Instantaneous f_c can be graphed against the horn length. A spreadsheet was designed with the appropriate equations to perform this analysis. Figure 4 shows how this method compares an exponential horn, a conical horn and an axisymmetric Quadratic Throat waveguide. For this comparison the throat entry and horn length are the same for all three. The mouth exit is the same for the exponential and conical horns. The mouth of the QT waveguide is slightly smaller than the other horns. It is smaller by the exact dimension of the throat entry. This is due to the geometry of this type of horn.

As would be expected, since its flare rate is the reference, the exponential horn has a constant value of f_c. The conical horn can be seen to have an initial value at the throat entry. This value steadily decreases until it reaches its minimum value at the mouth. This is indicative of the gradual decrease in the acoustic resistance of conical horns as frequency decreases.

The QT waveguide presents a very different loading characteristic. It has an initial value of 0 Hz at the throat. This can be attributed to the fact that at the throat entry the walls have no flare; they are normal to the plane of the throat entry. The value of f_c steadily increases until it reaches its maximum value at the point where the throat section joins the straight wall section. From this point it steadily decreases in the exact manner that the conical horn does. This gives us a very good indication as to the loading properties of the QT waveguide. While it is not quite as good as an exponential horn, it is almost so.

The development of the proper wavefront shape is of paramount importance for the directivity response to be as intended. The requirement that must be met for proper development is for the wavefront to remain normal to the waveguide boundary at all points along the boundary[2]. This means the waveguide must effectively transform the planar wavefront, presented to the throat entry by the driving unit, to a spherical wavefront. Figure 5 illustrates how this is accomplished. The wavefront is shown at regularly spaced intervals to depict its transformation and development. The spherical wavefront is achieved at the point where the waveguide transitions from its throat section to its straight wall section. From this point forward, the spherical wavefront progresses toward the mouth of the waveguide. The radius of curvature of the wavefront in and beyond the straight wall section is equal to its distance from the center of the throat entry.

It was observed that for a number of different throat entry diameters and design coverage angles, the size of the aperture between the two sections of the waveguide was near its optimum size according to Keele's prior work[4, 7]. This is not unexpected given the geometry of the design of the waveguide boundaries. This can be illustrated by examining a 60° x 40° waveguide with a maximum instantaneous f_c of 500 Hz. For an optimized aperture over the frequency range of f_c to $10f_c$, Keele's $k_c a_m$ value is .403. The $k_c a_m$ value for the QT waveguide described above is .424.

5. Performance evaluation of the new design

Two existing horns were redesigned using the QT waveguide technique. The new waveguides had the same coverage angles and walls in the outer section as the existing horns. The only major difference being that of the initial throat section. The first existing horn is that of a .875" entrance into an exponential throat section. This throat section joins to a straight wall section. It is a conventional, radial, constant directivity horn design. The second existing horn is that of a 2.0" entrance into a straight wall section. This section is maintained until the last ¼ of the horn length. At this point secondary flanges are added to minimize beamwidth narrowing prior to the horn losing its directivity control in the lower frequency region. It is a conventional straight wall horn. (It should be noted that the waveguide designed from this second existing horn was done with a 1.6" throat entry and not the original 2.0" entry. This new waveguide was being developed for a specific product and this smaller entry was more appropriate. The existing 2.0" entry horn has an adapter section that allows the same 1.6" exit driver to mount on it as well as the new waveguide. This smaller entrance may account for some of the increased output of the new waveguide as well as its improved directivity response in the very high frequency region. It otherwise should have no effect on the comparisons made for the purposes of the study.)

The new waveguides were extensively compared to the existing horn designs. Amplitude response, impedance, harmonic distortion at different power levels and directivity response measurements were made on the existing horns and the new waveguides. The same drive units were used for each comparable horn so as to minimize any measurement errors. Graphs of these measurements for the .875" throat entry devices are shown in figures 6 – 9. Graphs of these measurements for the larger throat entry devices are shown in figures 10 – 13. All of the measurements in these graphs are shown with 1/3 octave smoothing.

Figure 6 A is the amplitude response, B is the phase response of the two devices. The two devices are comparable in this area. Figure 7 shows the horizontal and vertical beamwidth measurements. The horizontal beamwidth of the two is almost identical. However, the vertical beamwidth of the QT waveguide is much closer to its intended coverage angle than the conventional design. Figures 8 & 9 present 2nd & 3rd harmonic distortion, respectively. It is clearly obvious that the QT waveguide has lower distortion. The 2nd harmonic distortion is 3.5 to 4 dB lower while 3rd harmonic distortion is reduced in excess of 9 dB over the conventional horn.

Figure 10 shows the amplitude response of the two large throat entry devices. The QT waveguide has an increase in the on-axis output above 5 kHz. At least part of this is attributable to the horizontal directivity in the same frequency region, shown in figure 11. The conventional horn's energy is being spread over a wider beamwidth above 5 kHz. The 2nd harmonic distortion graphs in figure 12 shows no real difference in the two devices. The QT waveguide actually has an average of 0.16 dB less to 0.4 dB more distortion than the conventional horn. Figure 13 reveals an overall decrease in the 3rd harmonic distortion of the QT waveguide.

Since, for the larger entry devices, the throat entry is larger for the conventional design, one would expect that its distortion would be lower. However, due to the fact that its distortion is marginally lower (2nd) to slightly higher (3rd) it is assumed that, had the throat entry sizes been equal, the same amount of reduction in distortion would have been realized as that of the .875" entry devices.

6. Conclusions

It can be surmised that the Quadratic Throat waveguide has a large reduction of 3rd harmonic distortion, while having a significant, yet smaller, reduction of 2nd harmonic distortion when compared to a conical horn or a conventional constant directivity horn comprised of an exponential section followed by a conical section.

Due to the design of the throat section, this new waveguide will admit and propagate a one-parameter wave when it is driven at its throat entry by a plane wave. The astigmatism typically found in conventional horns is eliminated in this new type of waveguide. This has a definite advantage when multiple horns are employed in an array as the apparent apex of the wavefront is in the same place for any given orientation of the waveguide. This feature makes spatial alignment of the individual elements in the array much easier. Once placed in the array these waveguides may be rotated, pitched or yawed as needed without affecting the spatial orientation of the wavefront, as it is truly spherical. With conventional horns, a change in orientation results in a change in the orientation of the wavefront curvature, as it is not spherical.

7. Acknowledgments

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8. References

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6. David Gunness, "Instantaneous Flare Rate", notes from Synergetic Audio Concepts - Loudspeakers Workshop, June 1997
7. D. B. Keele, Jr., "Optimum Horn Mouth Size", presented at the 46th AES Convention, Sept. 1973

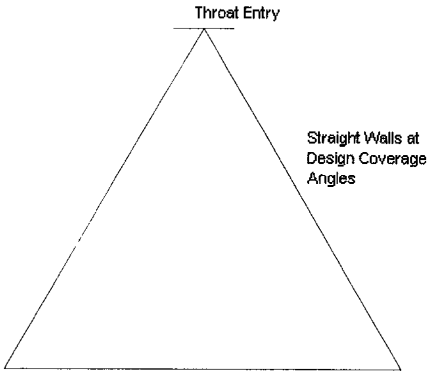


Figure 1

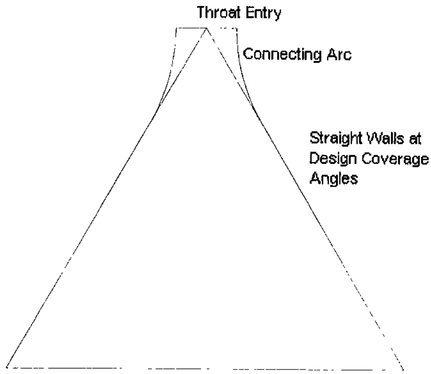


Figure 2

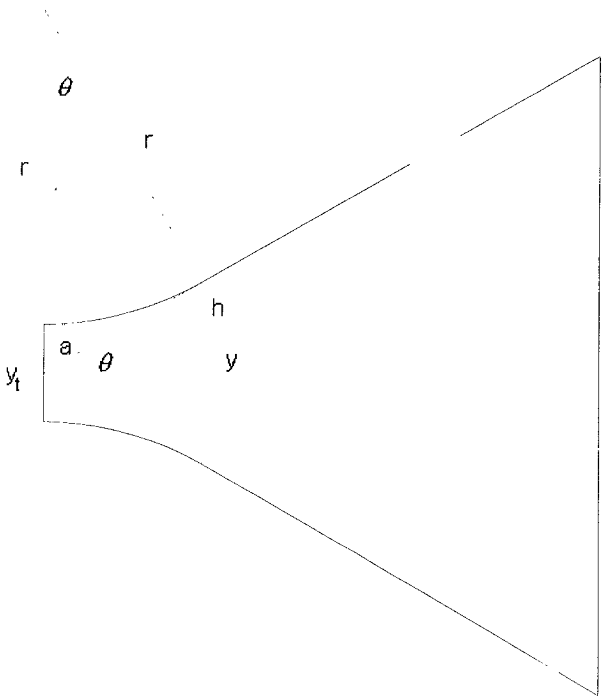


Figure 3:
Geometry of Quadratic
Throat waveguide
construction.

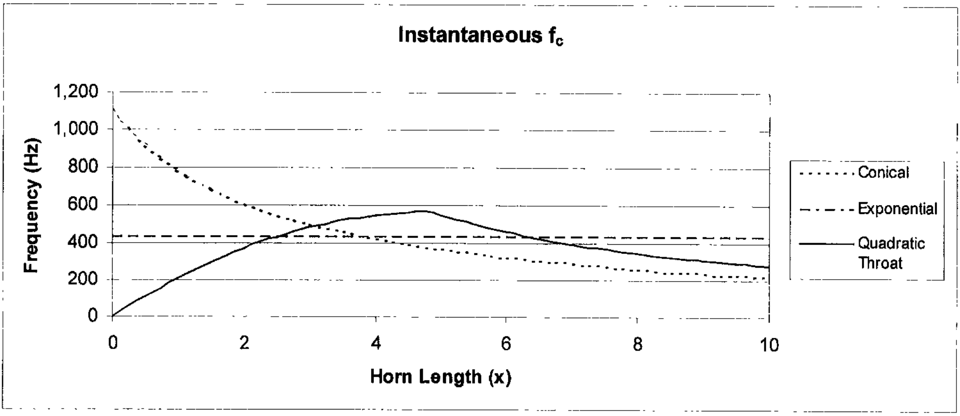


Figure 4

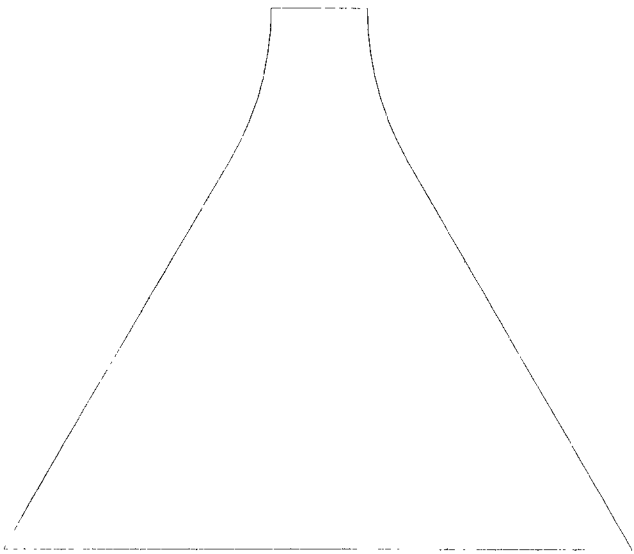
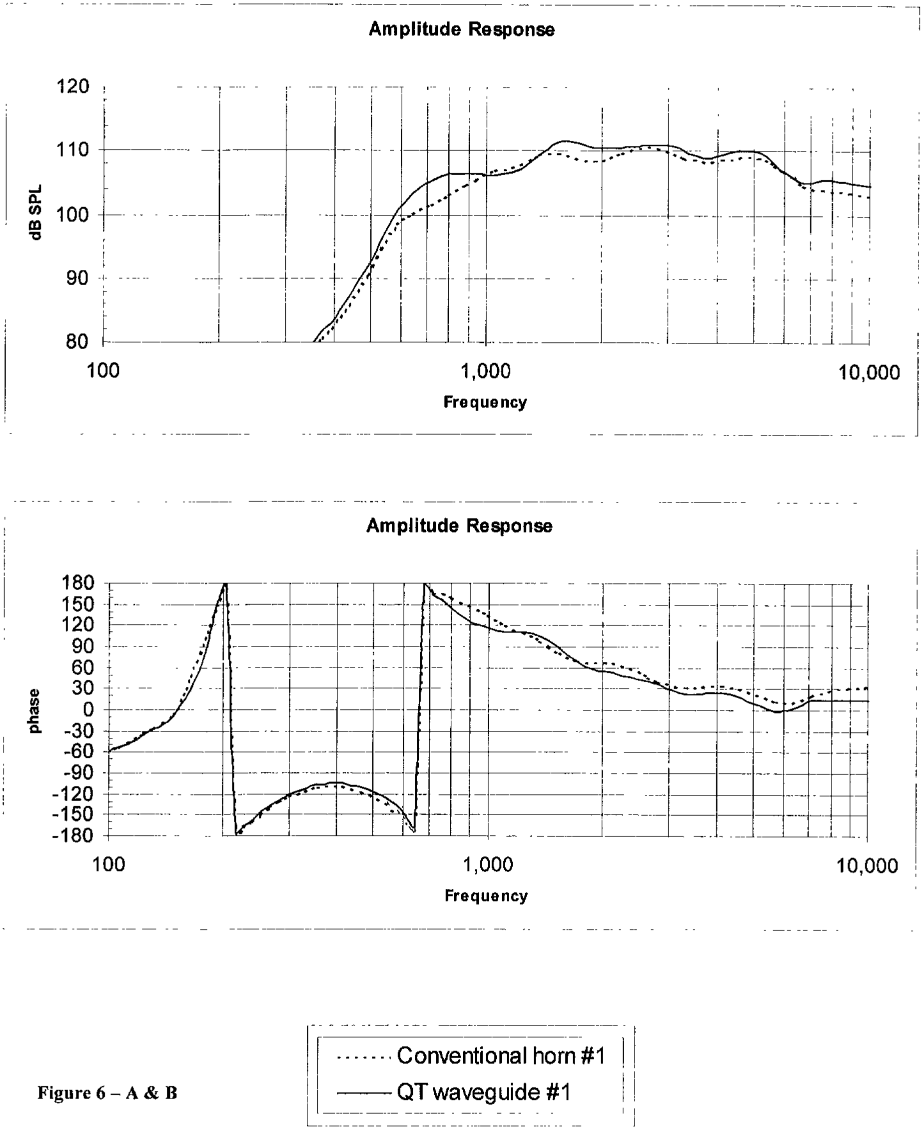


Figure 5:
Wavefront development inside waveguide. Note that the wavefront is always normal to the waveguide boundary.



Amplitude Response
dB SPL
120
110
100
90
80
100
1,000
10,000
Frequency
Amplitude Response
phase
180
150
120
90
60
30
0
-30
-60
-90
-120
-150
-180
100
1,000
10,000
Frequency
Conventional horn #1
QT waveguide #1

Figure 6 – A & B

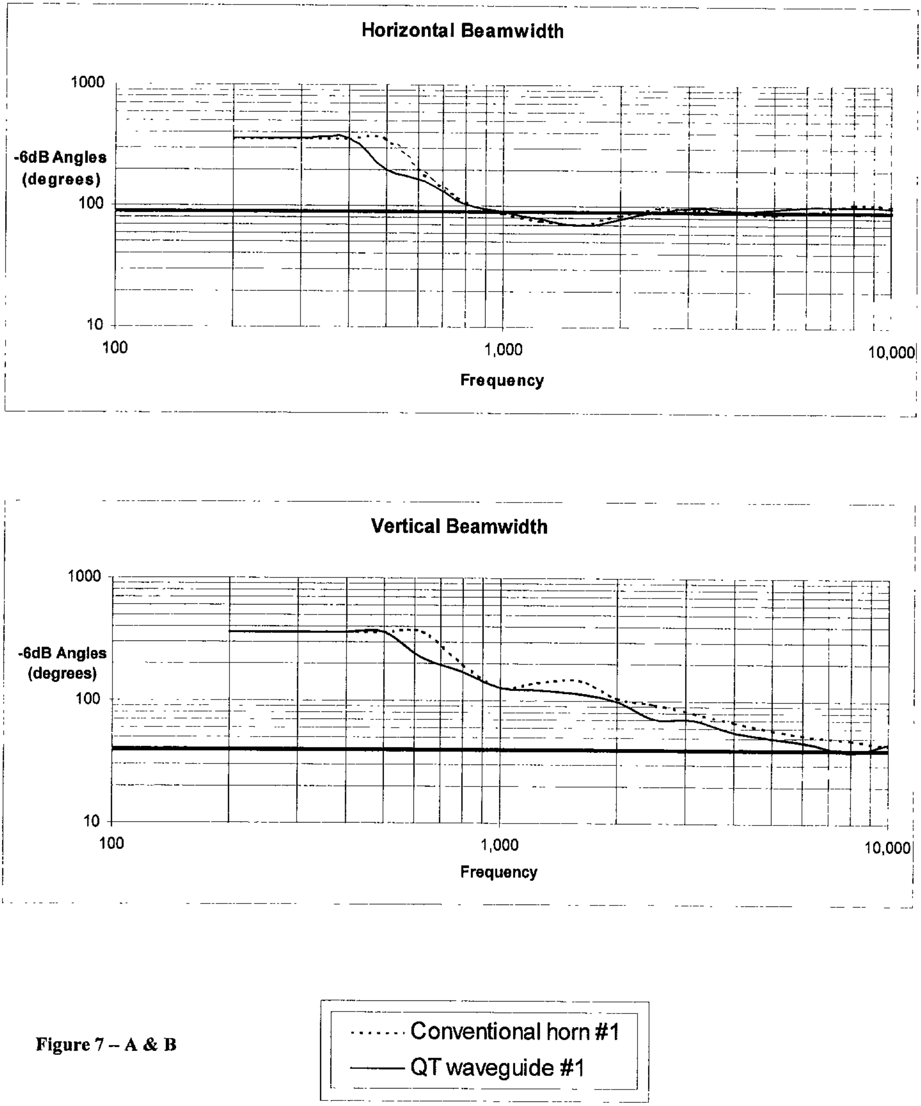
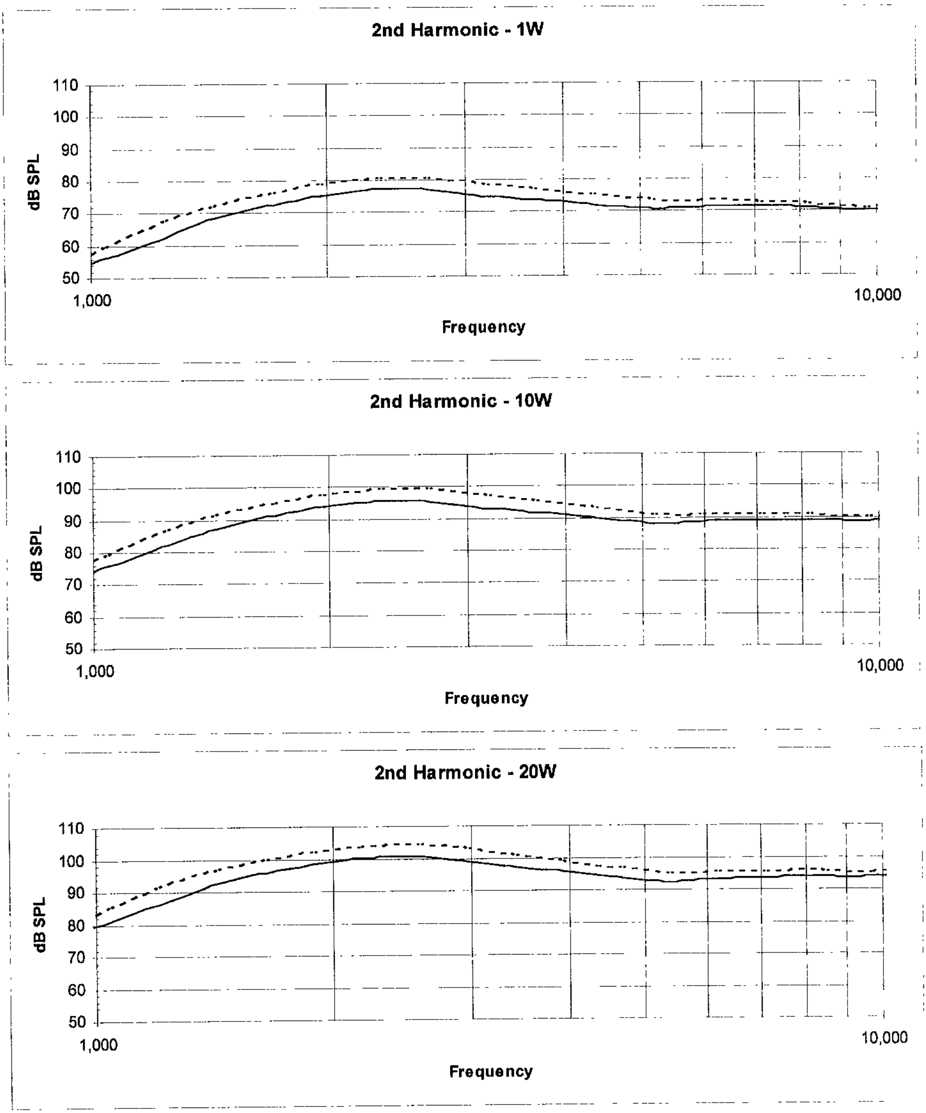

Horizontal Beamwidth
-6dB Angles (degrees)
1000
100
10
100
1,000
10,000
Frequency
Vertical Beamwidth
-6dB Angles (degrees)
1000
100
10
100
1,000
10,000
Frequency
Conventional horn #1
QT waveguide #1

Figure 7 – A & B


2nd Harmonic - 1W
110
100
90
80
70
60
50
dB SPL
1,000
10,000
Frequency
2nd Harmonic - 10W
110
100
90
80
70
60
50
dB SPL
1,000
10,000
Frequency
2nd Harmonic - 20W
110
100
90
80
70
60
50
dB SPL
1,000
10,000
Frequency

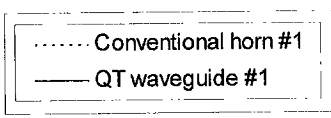

Conventional horn #1
QT waveguide #1

Figure 8 – A, B & C

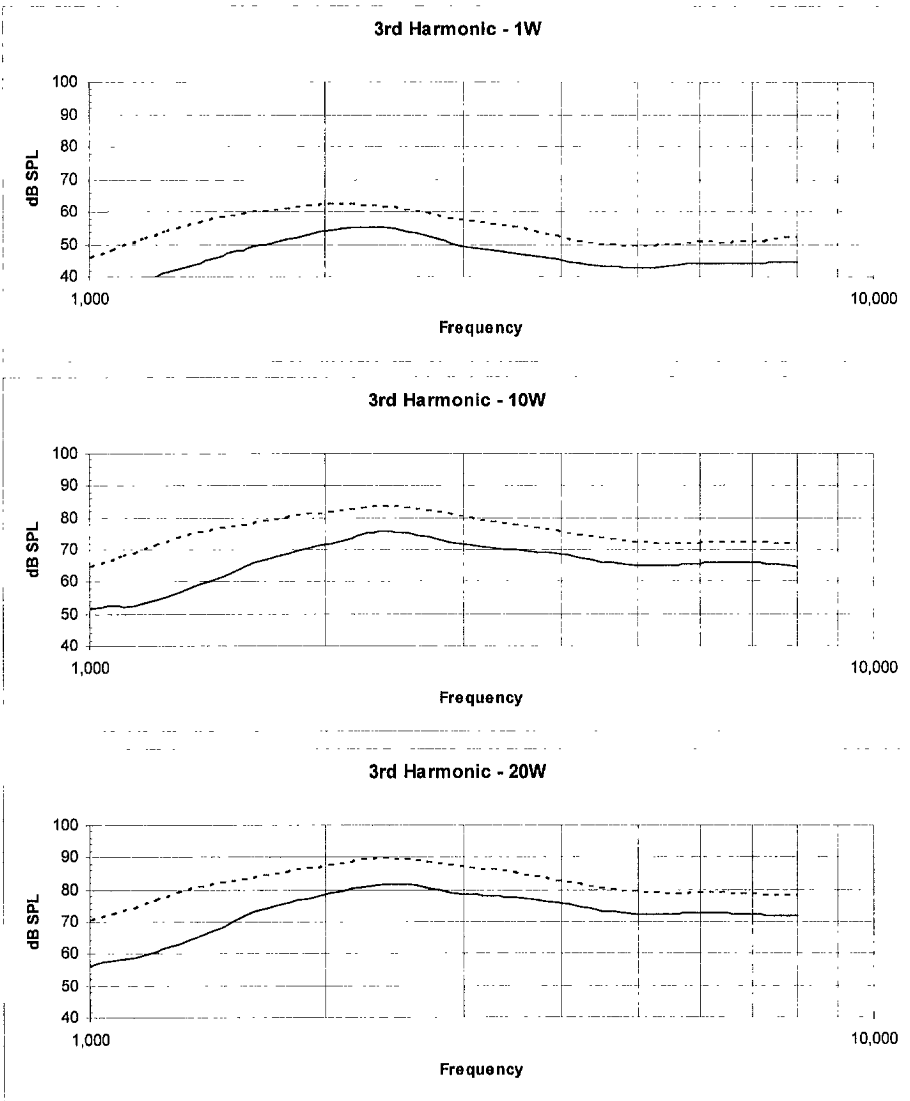

3rd Harmonic - 1W
dB SPL
100
90
80
70
60
50
40
1,000
10,000
Frequency
3rd Harmonic - 10W
dB SPL
100
90
80
70
60
50
40
1,000
10,000
Frequency
3rd Harmonic - 20W
dB SPL
100
90
80
70
60
50
40
1,000
10,000
Frequency
Conventional horn #1
QT waveguide #1

Figure 9 – A, B & C

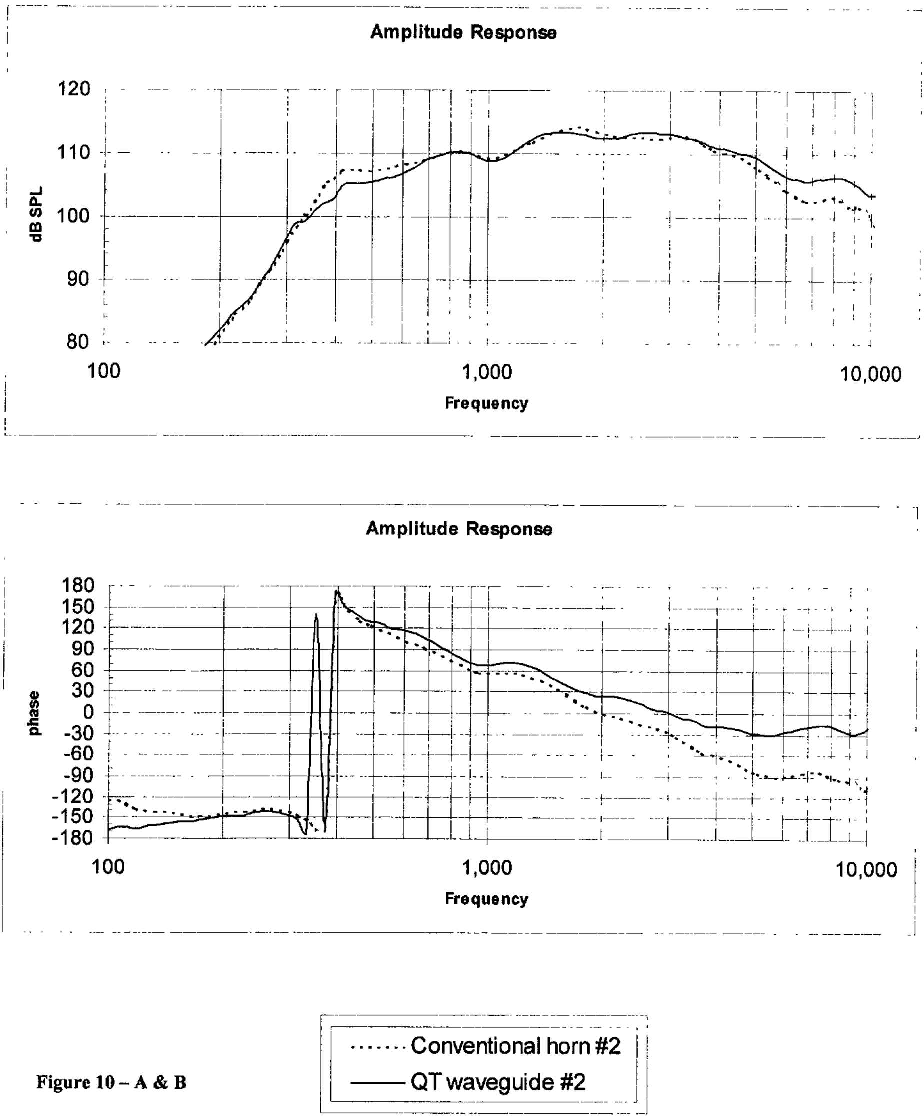

Amplitude Response
120
110
100
90
80
dB SPL
100
1,000
10,000
Frequency
Amplitude Response
180
150
120
90
60
30
0
-30
-60
-90
-120
-150
-180
phase
100
1,000
10,000
Frequency
Conventional horn #2
QT waveguide #2

Figure 10 – A & B

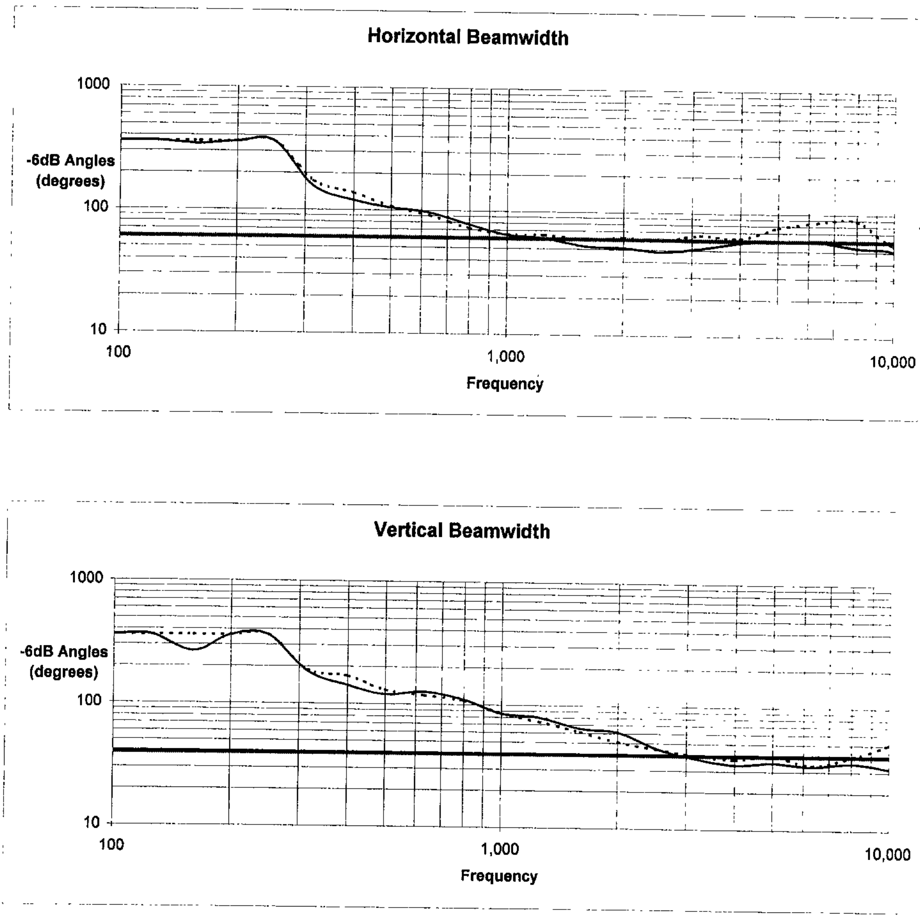

Horizontal Beamwidth
1000
-6dB Angles (degrees)
100
10
100
1,000
10,000
Frequency
Vertical Beamwidth
1000
-6dB Angles (degrees)
100
10
100
1,000
10,000
Frequency
Conventional horn #2
QT waveguide #2

Figure 11 – A & B

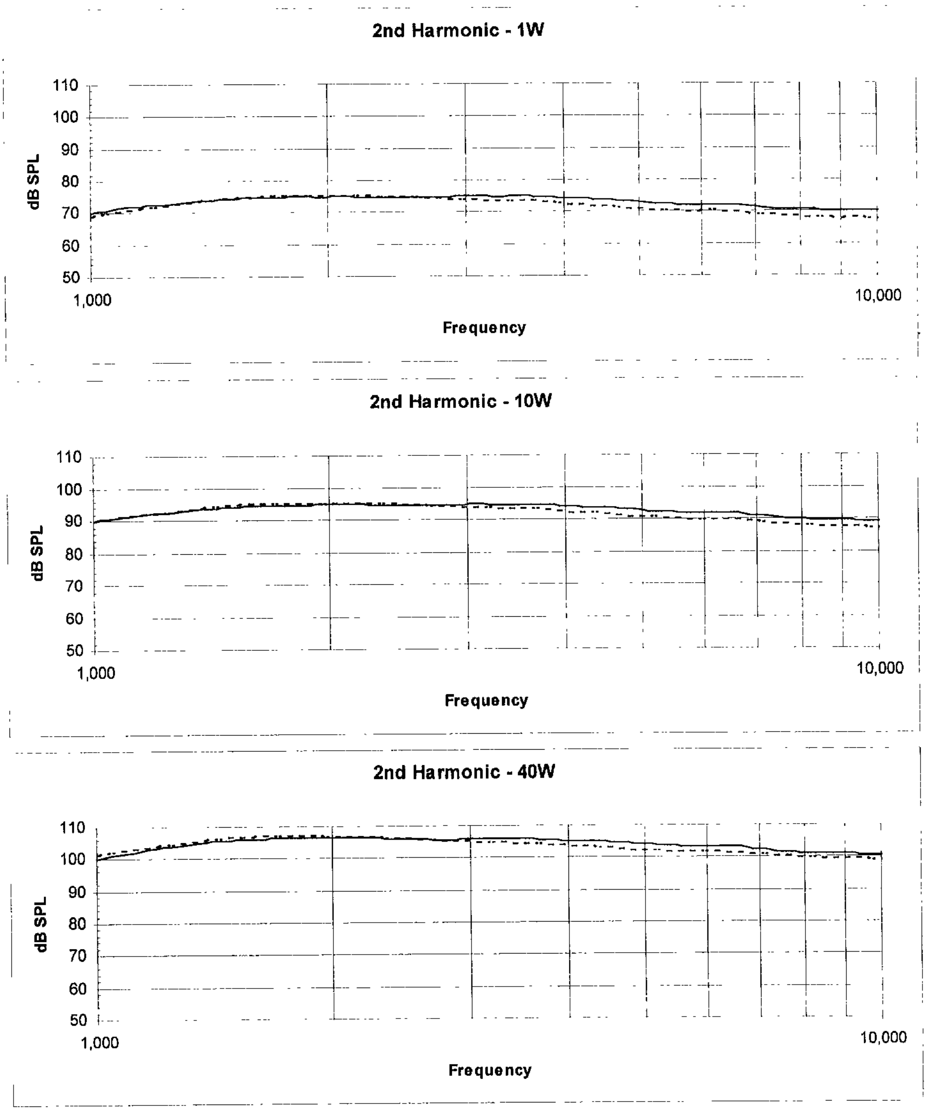
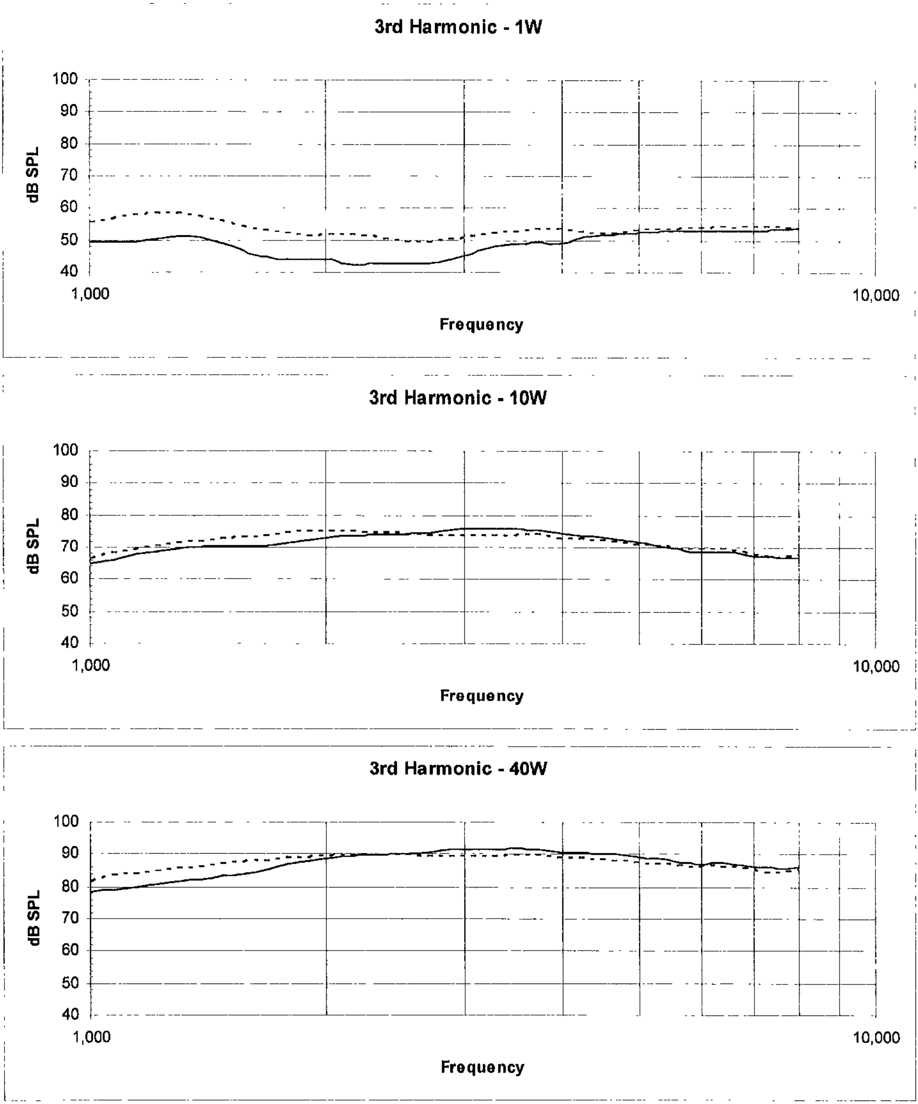


Figure 12 – A, B & C



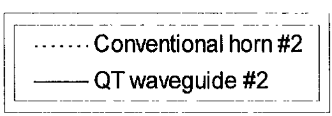


Figure 13 – A, B & C